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Evaluating Blocking Probability In Series Queue with Multi Server Stations

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Abstract— Seriesqueueswith blocking are widely used in mathematical modeling of computer and telecommunication system, emergency health care units. Although there are various publications on series queues with blocking, all models deal with single server in each station. In this paper we presented a two station tandem queue model with 2 servers in first station and c servers in second station and obtained the stationary blocking probabilities.

Index Terms—Tandem queue, Muti server stations, Blocking, Steady state solution.

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1 Introduction

A Lot of literature is available for two station tandem queues (one server in each station). A two server tandem queue with no intermediate waiting space was discussed by Avi – Itzhak, B., Yadin M [1]. N. U. Prabhu[9] discussed the transient analysis of tandem queue. Neuts [8] derived the steady state probabilities of two queues in series with finite intermediate waiting room.Langaris and Conolly[6] further extended this tandem queues for three stations with blocking. Zhu [12] analyzed a tandem queue with group arrivals and no intermediate buffer.Kness [5] discussed the concept of diffusion for tandem queues with general renewal input. Winfried and Steve[11] derived tandem queue with blocking with the new method. Changing the service pattern as phase type, tandem queues were derived by Valentina et al [10]. Concept of feed back is added to tandem queue with blocking by CohandraSekhar et al [2]. Tandem queue with blocking has been introduced in queue in queue networks by Dallery and Frein [3].

In this paper, a series queue of two stations (S1 and S2) has been considered. Each station has multiple servers in which the first station S1 has two servers in parallel service whereas the second server S2 has n parallel servers. Each arriving customer must go to both the service stations one after the other. Choice of choosing a server in each station is based on their availability if the servers are busy and is random if they are idle. There is no waiting space between the service stations. Therefore a customer who completes his service in S1 will wait at S1 itself if he sees all the n servers in S2 are busy thus blocking the server at S1 till one of the servers in S2 becomes free.

The rest of the paper is organized as follows. Mathematical formulation of the model is done in section 2. Section 3 contains the steady state solution. Section 4 contains a special case for the number of servers in each station. Section 5 is a numerical example of the model with two servers in station 1 and two servers in station 2. The conclusions and future work are given in section 6. Finally the references are listed.

2 Mathematical model

Let there be two servers in S_1 and hence the number of customers in S_1 is either 0(no customer) or 1(one in service) or 2(two in service) or 1*(only one customer and he is blocked) or 2*(two customers with one in service and one blocked) or 2** (two customers both blocked).

Let there be c servers in S_2. Thus the number of customers in S_2 is either 0 or 1or 2…or c. The queuing process is characterized by two random variables X_1 = {0, 1, 2, 1*, 2*, 2**} and X_2={0, 1, 2…c}.

The possible states of the system are {(0, 0),(0, 1)…(0, c), (1,0), (1,1)…(1, c),(2 ,0), (2, 1)...(2, c), (1*, c), (2*, c), (2**, c)}. Define the probability function as $P_{i,j}(t) = P\{X_1(t)=i, X_2(t)=j\}$.

The quasi birth death equations are given by,

$$P_{i,j}'(t) = A\,P_{i,j}(t) \qquad (1)$$

where A is a square matrix of order 3c+6.

A =

D1	D2	D3	D4
D5	D6	D7	D8
D9	D10	D11	D12
D13	D14	D15	D16

Where

D1	(0, 0)	(0, 1)	(0,2)	…	(0, c)
(0, 0)	$-\lambda$	0	0	…	0
(0, 1)	μ_2	$-\mu_2-\lambda$	0	…	0
(0,2)	$2\mu_2$	μ_2	$-3\mu_2-\lambda$	…	0
⋮	⋮	⋮	⋮	…	⋮
(0, c)	$c\mu_2$	$(c-1)\mu_2$	$(c-2)\mu_2$	…	$-c(c+1)\mu_2/2 - \lambda$

D2	(1, 0)	(1, 1)	(1,2)	…	(1, c)
(0, 0)	λ	0	0	…	0
(0, 1)	0	λ	0	…	0
(0,2)	0	0	λ	…	0
⋮	⋮	⋮	⋮	…	⋮
(0, c)	0	0	0	…	λ

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D3	(2, 0)	(2, 1)	(2,2)	…	(2, c)
(0, 0)	0	0	0	…	0
(0, 1)	0	0	0	…	0
(0,2)	0	0	0	…	0
⋮	⋮	⋮	⋮	…	⋮
(0, c)	0	0	0	…	0

D4	(1*, c)	(2*, c)	(2**,c)
(0, 0)	0	0	0
(0, 1)	0	0	0
(0,2)	0	0	0
⋮	⋮	⋮	⋮
(0, c)	0	0	0

D5	(0, 0)	(0, 1)	(0,2)	…	(0, c)
(1, 0)	0	μ_1	0	…	0
(1, 1)	0	0	μ_1	…	0
(1,2)	0	0	0	…	0
⋮	⋮	⋮	⋮	…	⋮
(1, c)	0	0	0	…	0

D6	(1, 0)	(1, 1)	(1,2)	…	(1, c)
(1, 0)	$-\lambda - \mu_1$	0	0	…	0
(1, 1)	μ_2	$-\mu_1 - \mu_2 - \lambda$	0	…	0
(1,2)	$2\mu_2$	μ_2	$-3\mu_2 - \lambda$	…	0
⋮	⋮	⋮	⋮	…	⋮
(1, c)	$c\mu_2$	$(c - 1)\mu_2$	$(c-2)\mu_2$	…	$-\frac{c(c+1)\mu_2}{2} - \lambda - \mu_1$

D7	(2, 0)	(2, 1)	(2,2)	…	(2, c)
(1, 0)	λ	0	0	…	0
(1, 1)	0	λ	0	…	0
(1,2)	0	0	λ	…	0
⋮	⋮	⋮	⋮	…	⋮
(1, c)	0	0	0	…	λ

D8	(1*, c)	(2*, c)	(2**,c)
(1, 0)	0	0	0
(1, 1)	0	0	0
(1,2)	0	0	0
⋮	⋮	⋮	⋮
(1, c)	μ_1	0	0

D9	(0, 0)	(0, 1)	(0,2)	(0, 3)	…	(0, c)
(2, 0)	0	0	$2\mu_1$	0	…	0
(2, 1)	0	0	0	$2\mu_1$	…	0
(2,2)	0	0	0	0	⋱	0
⋮	⋮	⋮	⋮		…	⋮
(2, c)	0	0	0		…	0

D10	(1, 0)	(1, 1)	(1, 2)	…	(1, c-1)	(1, c)
(2, 0)	0	μ_1	0	0	…	0
(2, 1)	0	0	μ_1	0	…	0
(2,2)	0	0	0	⋱	0	0
⋮	⋮	⋮	⋮	⋮	μ_1	⋮
(2, c)	0	0	0	0	…	0

D11	(2, 0)	(2, 1)	(2, 2)	…	(2, c)
(2, 0)	$-3\mu_1$	0	0	0	0
(2, 1)	μ_2	$-3\mu_1 - \mu_2$	0	0	0
(2,2)	$2\mu_2$	μ_2	$-\mu_1 - 3\mu_2$		0
⋮	⋮	⋮	⋮	⋮	⋮
(2, c)	$c\mu_2$	$(c-1)\mu_2$	0	μ_2	$-\frac{c(c+1)\mu_2}{2} - 6\mu_1$

D12	(1*, c)	(2*, c)	(2**,c)
(1, 0)	0	0	0
(1, 1)	0	0	0
(1,2)	0	0	0
⋮	⋮	⋮	⋮
(1, c-1)	$2\mu_1$	0	0
(1, c)	0	μ_1	$2\mu_1$

D13	(0, 0)	(0, 1)	(0,2)	…	(0, c)
(1*, c)	0	$c\mu_2$	$(c-1)\mu_2$	…	μ_2
(2*, c)	0	0	0	…	0
(2**,c)	0	0	0	…	$2\mu_2$

D14	(1, 0)	(1, 1)	(1,2)	…	(1, c)
(1*, c)	0	0	0	…	0
(2*, c)	0	$c\mu_2$	$(c-1)\mu_2$	…	μ_2
(2**,c)	0	0	0	…	$2\mu_2$

D15	(2, 0)	(2, 1)	(2,2)	…	(2, c)
(1*, c)	0	0	0	…	0
(2*, c)	0	0	0	…	0
(2**,c)	0	0	0	…	0

D16	(1*, c)	(2*, c)	(2**,c)
(1*, c)	$-c(c+1)\mu_2/2 - \lambda$	λ	0
(2*, c)	0	$-c(c+1)\mu_2/2 - \mu_1$	μ_1
(2**,c)	μ_2	0	$-3\mu_2$

3 STEADY STATE SOLUTION

The steady state solution of equation (1) is given by

$$A\,P_{i,j} = 0 \tag{2}$$

along with the normalisation condition

$$\sum P_{i,j} = 1 \tag{3}$$

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Equations (2) and (3) represent simultaneous equations in 3c+6 variables. Solving them we can find the steady state prb-abilities from which other measures that portray the system can be found.

4 SPECIAL CASE

Consider the case in which the number of servers in both stations are limited to two.

Since there are 2 servers in second station the various states are {(0, 0),(0, 1), (0, 2), (1,0), (1,1), (1, 2),(2 ,0), (2, 1), (2, 2), (1*, 2), (2*, 2), (2**, 2)}.

The steady state balance equations become

$$\lambda P_{00} = \mu_2 P_{01} + 2\mu_2 P_{02}$$
$$(\mu_2 + \lambda) P_{01} = \mu_2 P_{02} + \mu_1 P_{10} + 2\mu_2 P_{1^*2}$$
$$(3\mu_2 + \lambda) P_{02} = \mu_1 P_{11} + 2\mu_1 P_{20} + \mu_2 P_{1^*2} + 2\mu_2 P_{2^{**}2}$$
$$(\lambda + \mu_1) P_{10} = \lambda P_{00} + \mu_2 P_{11} + 2\mu_2 P_{12}$$
$$(\lambda + \mu_1 + \mu_2) P_{11} = \lambda P_{01} + \mu_2 P_{12} + \mu_1 P_{20} + 2\mu_2 P_{2^*2}$$
$$(\lambda + \mu_1 + 3\mu_2) P_{12} = \lambda P_{02} + \mu_1 P_{21} + \mu_2 P_{2^*2}$$
$$3\mu_1 P_{20} = \lambda P_{10} + \mu_2 P_{21} + 2\mu_2 P_{22}$$
$$(3\mu_1 + \mu_2) P_{21} = \lambda P_{11} + \mu_2 P_{22}$$
$$(3\mu_1 + 3\mu_2) P_{22} = \lambda P_{12}$$
$$(3\mu_2 + \lambda) P_{1^*2} = \mu_1 P_{12} + 2\mu_1 P_{21} + \mu_2 P_{2^{**}2}$$
$$(3\mu_2 + \mu_1) P_{2^*2} = \mu_1 P_{22} + \lambda P_{1^*2}$$
$$3\mu_2 P_{2^{**}2} = 2\mu_1 P_{22} + \mu_1 P_{2^*2}$$

5 NUMERICAL EXAMPLE

Customers arrive at a rate of 3 per minute to a healthcare system with two stations. Each station consists of two servers each. The service is done at a homogeneous rate 2 and 3 per minute respectively in each station. Find the blocking probabilities.

Given $\lambda = 3$
$\mu_1 = 2$
$\mu_2 = 3$

Solving these simultaneous equations arrived at section 4 with the normalisation $\sum P_{i,j} = 1$ and with the above values, we get

$P_{00} = 0.26$	$P_{01} = 0.12$	$P_{02} = 0.07$
$P_{10} = 0.24$	$P_{11} = 0.09$	$P_{12} = 0.02$
$P_{20} = 0.14$	$P_{21} = 0.03$	$P_{22} = 0.006$
$P_{1^*2} = 0.015$	$P_{2^*2} = 0.005$	$P_{2^{**}2} = 0.003$

6 CONCLUSION

Here the steady state solution of the model is discussed. Further the model can be treated for heterogeneous server. The solution in transient state can also be found. One can try to generalize the number of servers in station 1. Buffer can be introduced in between the two stations which is a most required model in practical life.

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